

Coordinate Path Expansion in Iterated (HD) Systems

Consider the iterated system

$$x_{t+1} = HDx_t$$

where H is a normalized Walsh–Hadamard transform and D is a diagonal sign matrix.

A useful and intuitive way to understand the dynamics is not through eigenspaces or global matrix multiplication, but through recursive coordinate propagation.

Following a Single Coordinate

At a single iteration step:

- D applies a local sign reflection,
- H redistributes each coordinate contribution globally.

One may therefore conceptually:

1. select a single coordinate contribution,
2. follow its propagation through one iteration,
3. then at the next iteration choose one resulting contribution again,
4. and recursively continue.

This produces a branching propagation tree.

After t iterations, each final coordinate receives contributions from a very large number of such propagation paths.

Combinatoric Growth of Paths

The output at depth t may therefore be viewed as a sum over branching coordinate histories:

$$x_i(t) = \sum_{\text{paths } p} \sigma(p) x_{j(p)}(0),$$

where:

- p indexes a coordinate propagation path,
- $\sigma(p)$ is the accumulated sign/interference factor,
- $j(p)$ identifies the originating coordinate.

The number of effective paths grows combinatorially with iteration depth.

Thus even though the global system remains:

- linear,
- orthogonal,
- finite-dimensional,

the decomposed causal structure becomes extremely large.

Refolding of the Branch Structure

An important feature is that the exponentially expanding propagation tree is repeatedly compressed back into the fixed-dimensional state vector.

Each iteration:

1. expands coordinate influence globally,
2. generates new interference pathways,
3. then refolds all accumulated contributions back into N coordinates.

The explicit combinatoric structure is therefore hidden inside a compact linear representation.

This repeated expansion-and-refolding process helps explain:

- transient interference structures,
- metastable oscillations,
- sensitivity to small changes in D ,
- and the apparent richness of iterated HD dynamics.

Relation to Path-Sum Intuition

This viewpoint has some intuitive similarity to path-sum descriptions used elsewhere in physics and applied mathematics, in the sense that:

- observable quantities arise from sums over many interfering propagation histories.

No deeper equivalence is implied here; the similarity is purely conceptual and geometric.

The important point is simply that:

- local observable behavior may be more naturally understood through recursive propagation histories than through conventional global spectral descriptions alone.
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Interpretive Value

The coordinate-path viewpoint is useful because it provides:

- a causal picture of information propagation,
- an intuitive explanation for interference complexity,
- and a human-understandable geometric interpretation of iterated HD dynamics.

Rather than viewing the system purely as “rotation in eigenspace,” one instead sees:

- recursive branching,
- interference accumulation,
- and continual global refolding of combinatorially many propagation paths.